REPORT No. 110

THE ALTITUDE EFFECT ON AIR SPEED INDICATORS

IN TWO PARTS

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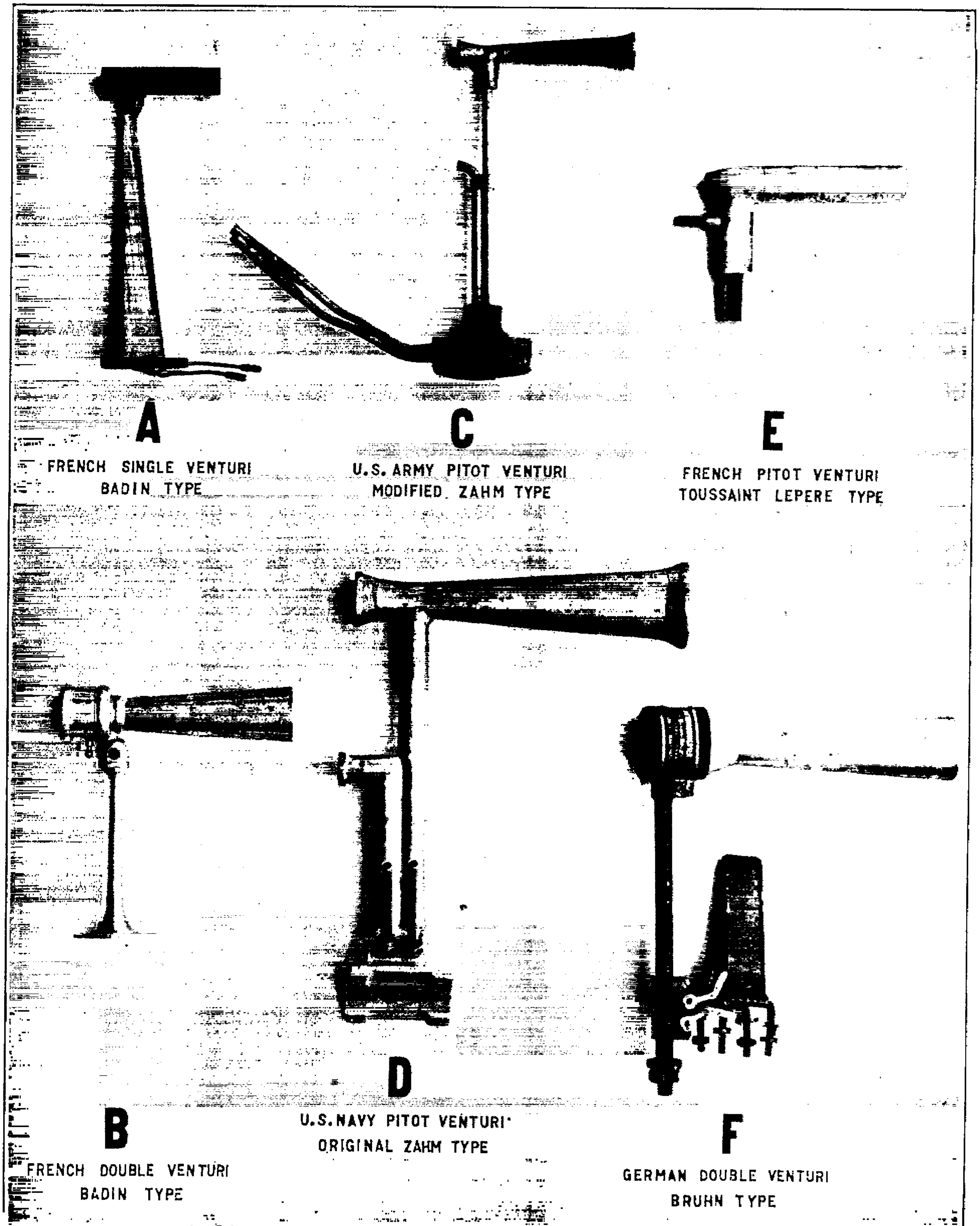

FIG. 1.—VENTURI TUBES.

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PART I.

THEORETICAL INTRODUCTION.

This report was begun in the fall of 1917 in connection with the testing of air speed indicators by the Bureau of Standards, and prepared at the request of the National Advisory Committee for Aeronautics.

1. OUTLINE OF INFORMATION REQUIRED.

In order to convert the readings of an air speed indicator, after correcting for ordinary instrumental errors, into true air speed, two steps are necessary:

(*a*) Arbitrary designation of standard atmospheric conditions near sea level.

(*b*) Determination of the effect on the performance of the instrument, of the departure from those conditions which may be met at any altitude.

In order to take the first step (*a*), it is necessary to have a complete enumeration of the conditions which influence the aerodynamic performance. It would not be enough to specify standard density, unless it were known that changes of density alone could modify the performance at a given speed.

To carry out the second step (*b*), it is necessary to secure, theoretically or experimentally, such information as would provide data for drawing up a complete set of curves connecting the performance of the instrument, on the one hand, with each of the variables governing its performance, on the other hand.

For example, in the case of the Pitot-Venturi type, a family of curves would be necessary connecting the differential pressure with speed, air density, air viscosity, and any other factors which appreciably alter the differential pressure.

Precisely similar information is desirable for all the other types of air speed indicators. These instruments, so far as aerodynamic performance is concerned, may be classified somewhat as follows:

1. Rotating surface type (Morell, etc.).
2. Direct impact type.
   - (*a*) With surface in a fixed direction (pressure plate, etc.).
   - (*b*) With variable direction of surface. (Pensuti, etc.)
3. Differential pressure type.
   - (*a*) Pitot tube (with static or with suction openings).
   - (*b*) Venturi tube (with or without static openings; single or double throat).
   - (*c*) Pitot-Venturi (Toussaint-Lepère; Zahm nozzle, U. S. Navy; modified Zahm nozzle, U. S. Army).
4. Air flow type (Prouty).
5. Nonmechanical types (hot wire, etc.).

Descriptive details of all these types are to be found in another paper.[1]

[1] General Report on Aeronautic Instruments, Investigation of Air Speed Indicators, by F. L. Hunt: National Advisory Committee for Aeronautics, to be published later.

This investigation was undertaken to supply, in some measure, the information outlined above as desirable. The extent to which such information was previously available is indicated in the following section dealing with the assumptions customarily made.

2. ASSUMPTIONS CUSTOMARILY MADE.

It has ordinarily been assumed that the density alone is sufficient to fix the standard atmospheric condition, (*a*), and that as regards departure from this condition, (*b*), the indication of the instrument at a given speed is directly proportional to the density for air speed indicators of the direct impact and differential pressure types and practically independent of density for indicators of the rotating vane type. In fact, the direct impact and differential pressure types are colloquially, though not scientifically, spoken of as "ρv^2" instruments and the rotating vane type as "true air speed" instruments.

The ρv^2 assumption is fairly satisfactory for direct impact and for Pitot tube instruments, but less so for Venturi tubes. In the case of the Pitot, it is recognized that a more accurate result can be deduced at the higher speeds by allowing for adiabatic compression of the air; and in the case of the Venturi, it is found by the present experiments that the viscosity of the air has to be taken into account at the lower speeds and higher altitudes.

Disregarding compressibility and viscosity, the correction for altitude is customarily made by the formula

$$v=\frac{v_i}{\sqrt{r}} \tag{1}$$

in which v is the true and v_i the indicated speed and r the relative density at the level in question, i. e.,

$$r=\frac{\rho}{\rho_0} \tag{2}$$

where ρ is the actual and ρ_0 the standard density. At 20,000 feet the relative density is about one-half, so for that altitude $v=v_i\sqrt{2}$ or the true speed is about 40 per cent greater than the indicated speed.

For graduating the dials to go with the Pitot-Venturi tubes of American manufacture, such as the United States Army tube illustrated in figure 1, the Zahm nozzle formula [2]

$$p=.00313\ v^2 \tag{3}$$

is employed, in which v denotes the speed in miles per hour corresponding to a differential pressure p inches of water.

This formula assumes for the standard air density 1.221×10^{-3} gms/cm^3, which is the density dry air would possess at a temperature of 16° C. under the normal sea level pressure of 760 mm. mercury.

3. LIMITATIONS OF DEDUCTIVE THEORIES.

By means of thermodynamic reasoning, treating the atmosphere as an ideal gas, it is possible to throw the Pitot tube formula into a more general one

$$p=\tfrac{1}{2}\ \rho v^2\ (1+C) \tag{4}$$

in which the correction term C stands for a certain complicated function which vanishes for a perfectly incompressible fluid, or for zero speed. For a speed of 110 miles per hour C is of the order of 0.5 per cent and should not be neglected if the most probable result is required; and yet so many simplifying assumptions have to be introduced in the derivation that the result is not entirely free from doubt.[3]

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[2] Strictly, $v=17.88\ \sqrt{p}$.

[3] E. Buckingham, on the Theory of the Pitot tube, Technical Report No. 2, National Advisory Committee for Aeronautics, 1915.

In just the same way a thermodynamic formula for the Venturi tube can be deduced, which attempts to take account of the geometrical shape of the tube and the compressibility of the air: but in this case the simplifying assumptions which have to be introduced are so appalling, that the result is only of academic interest. For example, it has to be assumed that the fluid is free from turbulence and at the same time without viscosity: and all disturbances outside of the tube have to be ignored. This means that the problem has to be treated like that of a continuous hydraulic pipe line, and that two instruments having the same interior channel would generate the same suction at the throat, no matter if one of them had a very much bigger bulge on the outside, such as to drag a lot of air along with it—a conclusion contrary to experience.

At best the thermodynamic formula could only hope to show the effect of compressibility, so far as properties of the fluid are concerned, since viscosity and turbulence are excluded at the start. Yet the experiments which will be reported here prove that the compressibility effect is a comparatively negligible one. Thus the Venturi tube problem is too complicated to be handled with advantage by purely deductive methods at present.

4. DIMENSIONAL THEORY.

A fruitful compromise between purely deductive theory on the one hand and interminable experimenting on the other hand, is made possible by dimensional reasoning.[4]

Consider, first, the Venturi tube. Let p denote the differential pressure (i. e., suction) generated at a speed v, the tube being pointed head on (i. e., without yaw) into a perfectly undisturbed medium (the atmosphere, for example, or water). It is understood that the speed v has remained constant for some moments so that a steady state is established. Let the mechanical properties of the medium be specified by its density ρ, viscosity μ, and compressibility modulus of elasticity E. On account of the rapid movements involved, it will be the adiabatic, not the isothermal, elasticity which is needed.

The three properties ρ, μ, and E are defined in the usual way. Thus the density is the mass per unit volume,

$$\rho \equiv \frac{M}{V} \tag{5}$$

The viscosity is the shearing stress per unit rate of shear, i. e.,

$$\mu \equiv \frac{f}{\left(\frac{dv}{dy}\right)} \tag{6}$$

in which f is the tangential force per unit area, or shearing stress, brought into play by distorting or shearing the fluid at such a rate that a velocity gradient $\frac{dv}{dy}$ is set up. Here dv is the difference in speed between the top and bottom surface of a layer dy units thick. The elasticity is the increase of hydrostatic pressure P per unit decrease of volume measured as a fraction of the volume V, i. e.,

$$E \equiv \frac{dP}{-\left(\frac{dV}{V}\right)} = -V\frac{dP}{dV} \tag{7}$$

The medium will be assumed homogeneous so that ρ, μ, and E have the same values at all points, both inside and outside the nozzle. This is admittedly an approximation, for the fluid is slightly warmer where most compressed, and so all of its constants are a trifle different at such a point. Finally let D stand for any agreed-upon linear dimension of the nozzle, as for example, the throat diameter.

Under these circumstances the differential pressure p evidently depends on the speed v, on the mechanical properties of the fluid, ρ, μ, and E, and on the absolute size D and the geo-

[4] Cf. E. Buckingham: Dimensional Theory of Wind Tunnel Experiments, Smithsonian Misc. Papers, v. 62-4, pp. 15-26, 1916; Model Experiments and the Form of Empirical Equations, Transactions A. S. M. E., v. 36, pp. 263-296, 1915.

metrical shape of the nozzle. Under the conception of shape are to be included the roughness of the surfaces and the contour of all adjacent parts that may cause disturbance. If no further physical quantities are apparent which can sensibly influence the phenomenon, then some relation

$$p = \text{funct}\ (v, \rho, \mu, E, D) \tag{8}$$

must exist, the specific form of which remains to be discovered by experiment, but which will be the same for all geometrically similar systems.

The object of our investigation, so far as Venturi tubes are concerned, consists in determining the form of that equation; and there will be an analogous equation for every other type of air speed indicator. It has to be done experimentally, but dimensional reasoning serves to simplify the planning of the experiments, and the interpretation of the observations.

Since equation (8) is physically complete, it must, when written out in full, have the same dimensions on both sides. The dimensions of the constituent quantities are as follows, taking mass (m), length (l), and time (t) for the necessary fundamental units:

$$\begin{aligned} p &: m\, l^{-1}\, t^{-2} \\ v &: l\, t^{-1} \\ \rho &: m\, l^{-3} \\ \mu &: m\, l^{-1}\, t^{-1} \\ E &: m\, l^{-1}\, t^{-2} \\ D &: l \end{aligned}$$

It can now be shown by the π-theorem (Buckingham, loc. cit.) or verified by inspection that the only form (8) can take, which will meet the requirement for dimensional homogeneity, is identical with, or reducible to, the general equation

$$p = \rho v^2\ \text{funct}\left(\frac{Dv\rho}{\mu},\ \frac{E}{\rho v^2}\right) \tag{9}$$

It will be useful to write out two modifications of this equation. The velocity of sound in a fluid, C, is given by the well-known expression

$$C = \sqrt{\frac{E}{\rho}} \tag{10}$$

Hence E, where it occurs in (9) above, can equally well be replaced by ρC^2, so (9) can be rewritten

$$\frac{p}{\rho v^2} = \varphi\left(\frac{Dv\rho}{\mu},\ \frac{C}{v}\right) \tag{11}$$

using the symbol ϕ to denote some unknown function of the *two* arguments, or independent variables, inside the parenthesis. This equation is of interest in connection with the water-channel experiments, to be described later.

Again, it is a well-known thermodynamic result that

$$E = \kappa P \tag{12}$$

for an ideal gas, κ being the specific heat ratio (about 1.4) and P the barometric pressure. (This relation follows from (7) in conjunction with the adiabatic compression equation $PV^{\kappa} = \text{const.}$) Replacing E by its equivalent κP in (9) gives

$$\frac{p}{\rho v^2} = \psi\left(\frac{Dv\rho}{\mu},\ \frac{\kappa P}{\rho v^2}\right) \tag{13}$$

The unknown functions ϕ and ψ are different, though they might have been kept identical by writing $\left(\frac{C}{v}\right)^2$ instead of $\frac{C}{v}$ in (11). This equation, (13), is of interest in connection with the observations in a wind stream at reduced barometric pressure, which remain to be described.

It is essential to realize that in equations (11) and (13) there are only *two* independent variables, not *five*, as in (8). Thus in equation (13) the dependent variable $\frac{p}{\rho v^2}$ is expressed as a function of the two independent variables $\frac{Dv\rho}{\mu}$ and $\frac{\kappa P}{\rho v^2}$. Readers not accustomed to this point of view may be helped by a change of notation: Write y for $\frac{p}{\rho v^2}$, x for $\frac{Dv\rho}{\mu}$, and z for $\frac{\kappa P}{\rho v^2}$ Then (13) becomes simply

$$y=\psi\ (x, z) \tag{14}$$

an ordinary surface in three coordinates. It is by such a surface (or family of plane curves) that the experimental observations ought to be depicted, instead of attempting to separate out the original quantities as in (8). They can be separated later, after the best possible empirical expression (14) has been fitted to the plotted points.

By means of the equations just deduced, especially (11) and (13), a comparatively economical program of experimental work can readily be laid out.

The foregoing analysis applies without change of notation to all differential pressure instruments of rigid shape. To extend it to direct impact instruments requires that the performance be expressed by F, the total force acting, instead of by the differential pressure p. Referring to equation (9), replace p by $\frac{F}{D^2}$ to preserve the dimensions unchanged and the result becomes

$$F=\rho v^2 D^2 \text{ funct}\left(\frac{Dv\rho}{\mu},\ \frac{E}{\rho v^2}\right) \tag{15}$$

This is the general equation for a pressure-plate air speed indicator. Except for extraordinarily low speeds the viscosity can not enter very seriously, consequently as an approximation which is safer the higher the speed,

$$F=\rho v^2 D^2 \text{ funct}\left(\frac{E}{\rho v^2}\right) \tag{16}$$

For since $\frac{Dv\rho}{\mu}$ is a *single* argument, if a change in viscosity causes no change in force, nothing else that controls the magnitude of $\frac{Dv\rho}{\mu}$ can do so either; hence the whole argument drops out. For speeds below, say, 150 miles an hour, where there is not much compression, (16) reduces to

$$F=\text{const}\times\rho v^2 D^2, \tag{17}$$

an example of the ρv^2 law. The constant is the same for pressure plates of different sizes provided they are strictly geometrically similar in all essential parts—including the sharpness of the edges, and proximity of the connections—and also provided, as was stated in the beginning, that the instrument is moving head on into an undisturbed atmosphere. Probably these conditions can be more easily fulfilled for pressure plates, and for Pitot tubes, than they can for Venturi tubes.

When the direction of the surface is not fixed, but free to change under increasing force of impact subject to the control of a spring as in the Pensuti air speed indicator, the problem is not so simple. The stiffness of the spring, S (force per unit displacement), now enters as an additional variable, so that (15) has to be expanded into the form

$$F=\rho v^2 D^2 \text{ funct}\left(\frac{Dv\rho}{\mu},\ \frac{E}{\rho v^2},\ \frac{S}{\rho v^2 D}\right) \tag{18}$$

In the ordinary case where viscosity and compressibility are negligible this reduces to

$$F=\rho v^2 D^2 \text{ funct}\left(\frac{S}{\rho v^2 D}\right) \tag{19}$$

instead of to (17). Some interesting conclusions applicable to the Pensuti and similar instruments can at once be drawn from this equation.

Let X stand for the new argument $\frac{S}{\rho v^2 D}$. Then $f(X)$ must be some function which starts at the origin and approaches asymptotically a maximum value, for an infinitely stiff spring, equal to the constant of equation (17); for equation (19) should reduce to (17) if the geometrical shape is constant. Hence in general the force (and therefore deflection) varies with speed less rapidly than the square, and with density less rapidly than the first power.

The altitude effect on instruments of the Pensuti class can now be deduced from observations made at sea level with varying speed. Suppose that a series of such observations gave

$$F \propto v^n \tag{20}$$

where n is some numerical value probably between 1 and 2. Then since (20) must be a special case of (19),

$$f(X) = X^{1-\frac{n}{2}} \tag{21}$$

$$\therefore\ F = \text{const} \times \rho^{\frac{n}{2}} v^n D^{1+\frac{n}{2}} S^{1-\frac{n}{2}} \tag{22}$$

whence

$$F \propto \rho^{\frac{n}{2}} \tag{23}$$

Thus the observation that the force varies with the nth power of the speed, leads by virtue of (19) to the inference that the force would vary with the $\frac{n}{2}$ power of the density, and with certain other powers of the size D and spring stiffness S.

By going back and differentiating equation (19), a still more general relation for the altitude effect in terms of the speed effect can be obtained, namely,

$$\frac{\partial F}{\partial \rho/\rho} = \frac{1}{2} \frac{\partial F}{\partial v/v} \tag{24}$$

This applies to all direct impact instruments operating over the intermediate range of speeds where neither viscosity nor compressibility have to be considered. Like other results afforded by dimensional reasoning, it is not limited by any assumption or restriction as to the geometrical complexity of the instrument or the irregularity of the motion set up in the fluid.

5. EXPERIMENTAL PROGRAM.

The experiments to be reported in this publication all relate to Venturi tubes and may be grouped as follows:

(a) Water channel experiments to determine the degree of dynamical similarity attainable between air and water, and to discover whether compressibility has to be taken into account.

(b) Observations in a wind stream at reduced pressure so as to determine the effect of density and viscosity by direct experiment.

(c) Airplane observations as a practical check on the foregoing laboratory results.

(d) Ordinary wind tunnel tests.

The need for these various experiments and the inferences possible from each can be readily seen in the light of the dimensional theory which has just been developed.

It was thought that for some purposes water channel observations on air speed nozzles would be more convenient than wind tunnel tests provided a reasonable degree of dynamical similarity proved attainable. From equation (11) neglecting compressibility the condition for similarity is found to be

$$\left(\frac{Dv\rho}{\mu}\right)_{\text{water}} = \left(\frac{Dv\rho}{\mu}\right)_{\text{air}}. \tag{25}$$

This condition can be fulfilled by towing the nozzle through the water at a speed slow enough to compensate for the relatively lower value which the kinematic viscosity $\frac{\mu}{\rho}$ has in water compared to air. Thus, denoting kinematic viscosity by ν, and distinguishing the water channel observations by primes, equation (25) reduces to

$$\frac{v'}{v}=\frac{\nu'}{\nu}\cdot\frac{D}{D'}. \tag{26}$$

At a temperature of 16° C. the kinematic viscosity of water is about one-thirteenth that of air; hence the corresponding speed in water would be about one-thirteenth of the speed in air provided the same nozzle is used so that $D'=D$. Under these corresponding conditions equation (9) shows that

$$\left(\frac{p}{\rho v^2}\right)_{\text{air}}=\left(\frac{p}{\rho v^2}\right)_{\text{water}} \tag{27}$$

from which

$$\frac{p}{p'}=\left(\frac{\rho}{\rho'}\right)\left(\frac{v}{v'}\right)^2. \tag{28}$$

Thus the differential pressure p which the nozzle would generate in air at a speed v can be predicted by observing the value p' realized in water at a speed v', provided the assumptions made are correct.

The most violent assumption made is that the effect of compressibility can be neglected in passing from such an incompressible fluid as water to such an easily compressible one as air. The value of E is about 20,000 times greater for water than for air. Thermodynamic theory suggests that the more readily compressible fluid should give the greater differential pressure at any one speed, and if the effect of compressibility is appreciable at all, it will be brought out in an exaggerated degree by the water channel experiments.

To test this assumption, observed values of $\frac{p}{\rho v^2}$ may be plotted as ordinates against $\frac{Dv\rho}{\mu}$ as abscissas, both for observations in air and in water. If the influence of compressibility is negligible the two curves, however irregular, ought to coincide. If they do not, then it is impossible to represent the results satisfactorily on a two-coordinate diagram; a third axis, for values of $\frac{C}{v}$, should be constructed and the data shown on a surface in space as implied by equation (11).

It turns out, as will be shown in detail later, that compressibility is practically but not wholly negligible. The agreement between wind tunnel and water channel observations is sufficient for predicting the order of magnitude of the differential pressure available from any tube of new design, and the water channel tests are also adequate for detecting small differences in the performance of nozzles produced in quantity from the same pattern.

If, however, the effect of varying the compressibility is found to be comparatively small when contrasting two media so different as air and water, then it will undoubtedly be negligible altogether where the use of the nozzle is confined to air alone. This is the most significant result of the water channel investigation, and warrants proceeding to the next series of experiments with attention directed to density and viscosity rather than to compressibility.

These next experiments were made in a wind stream at reduced pressure, the apparatus consisting of a small airtight tank, referred to as the vacuum wind tunnel. In this way air densities corresponding to various altitudes, and viscosities corresponding to various temperatures, could be realized in the laboratory. The results are plotted with $\frac{p}{\rho v^2}$ against $\frac{Dv\rho}{\mu}$ as before, since there is now no question of a third coordinate.

Clearly if the ρv^2 law holds, $\frac{p}{\rho v^2}$ will remain constant over the full range of conditions experienced; that is, the curve will be a horizontal straight line, parallel to the $\frac{Dv\rho}{\mu}$ axis. For a Pitot tube the ordinate of this line will be one-half, since $p=\frac{1}{2}\rho v^2$. For a Zahm nozzle performing in accordance with equation (3) the ordinate would be 3.2.

Any departure from horizontality not only signifies a departure from the ρv^2 law as regards the mathematical form in which ρ and v enter the law, but evidently also signifies that another physical quantity, viscosity, has begun to play a part. This complicates the altitude correction; for the viscosity as well as the density will be different at different atmospheric temperatures. It will be seen that the curve does have a considerable slope at low air speeds such as occur in the flight of dirigibles and the landing of airplanes, but not at the higher speeds.

This result was anticipated from the water channel experiments and also from another interesting circumstance. In developing the smaller modified Zahm nozzle for the Army it has been learned that it was not found possible to keep the new design geometrically similar to the original Zahm nozzle and still preserve the original calibration curve. This fact alone is evidence that viscosity makes a difference. For by (13) the size D can not enter unless the viscosity does also.

Having demonstrated the effect of viscosity and density under laboratory conditions, which had the advantage of direct control of the separate variables but the disadvantage of a restricted space not perfectly simulating free air conditions, it seemed worth while to proceed with airplane tests. This was done, and the results will be found plotted, as before, with $\frac{p}{\rho v^2}$ against $\frac{Dv\rho}{\mu}$. They agree qualitatively with the earlier laboratory results and afford more reliable numerical values, although not extending to such low densities.

Finally the results of ordinary wind tunnel tests on several different types of Venturi tubes are brought together for comparison. These too are reported by the dimensionless coordinate diagram, which is particularly well adapted for drawing inferences in regard to the altitude effect.

It was not feasible or necessary to make all the different kinds of tests on each of the tubes. The actual sequence followed is given below:

1. Water channel and wind tunnel tests on two French Venturi tubes, Badin type, one single and one double.
2. Vacuum wind tunnel tests on one American Pitot-Venturi tube, United States Army modified Zahm type.
3. Airplane flight tests on the foregoing Pitot-Venturi tube.
4. Study of ordinary wind tunnel data, taken at different times, on two of the foregoing tubes and on a French Pitot-Venturi, Toussaint Lepère type, and a German double Venturi, Bruhn type.

These tubes, together with the original Zahm nozzle, are shown in the photograph, figure 1.

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PART II.

EXPERIMENTS WITH VENTURI TUBES.

1. WATER CHANNEL EXPERIMENTS.

For the purpose of testing the two French Venturi tubes in water the 400-foot towing tank of the Bureau of Standards was placed at our disposal by Mr. W. F. Stutz, whose cooperation in this feature of the work is acknowledged. The nozzle under investigation was mounted about 40 cm. below the surface of the water on a rigid rod extending vertically down from the electric car which runs along a track over the tank. The speed of the car could be controlled and measured with an accuracy of the order of 1 or 2 per cent. The differential pressure was measured on a mercury manometer connected with the nozzle by water-filled

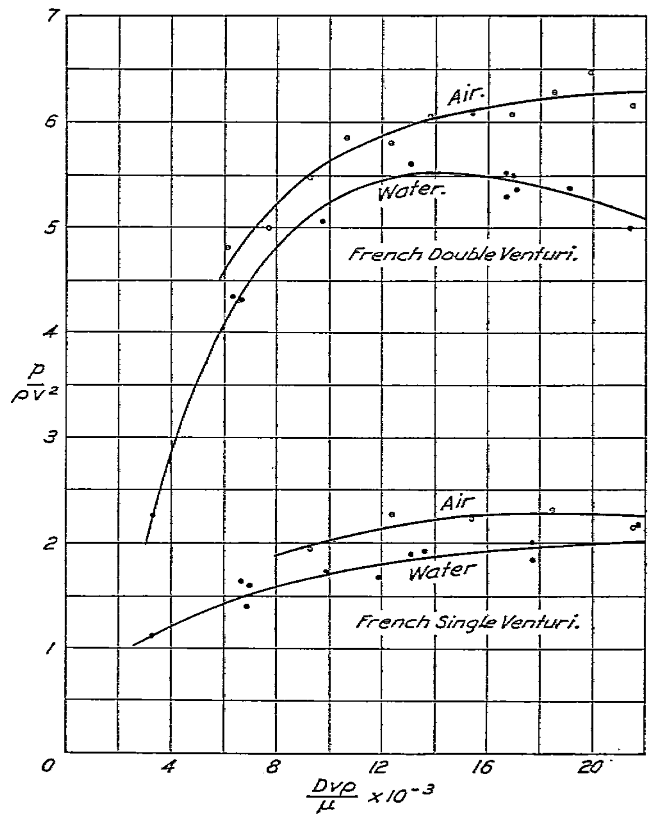


Fig. 2.—Water channel experiments.

tubing. At the high speeds and large suctions met in the last few observations on the double Venturi an error may exist due to time lag, of such a nature as to make the recorded values of the differential pressure too low. In those particular instances it was difficult to be sure that the manometer had risen to its maximum value in the short time interval available. Aside from this, it is thought unlikely that any important errors can have crept in. Due consideration was given to the necessity for avoiding turbulence and general movement of the water, and for keeping the depth of immersion sufficient to avoid surface disturbances.

Thus with a depth of immersion of 13 cm. one test at a speed of 4.77 miles per hour gave a differential pressure of 29.2 inches of water; a second test at the same speed gave 30.8 inches; while a third test at the same speed, upon increasing the depth to 40 cm., gave substantially the same result, 30.3 inches.

The following data were obtained for the French single Venturi (A, Fig. 1) in water and are plotted in fig. 2:

French single Venturi, in water.

Speed (miles per hour).	Differential pressure (inches water).	$\frac{p}{\rho v^2}$	$\frac{Dv\rho}{\mu}$
1.29	1.5	1.12	3,240
2.68	9.4	1.63	6,700
2.73	8.4	1.40	6,830
2.77	9.9	1.60	6,930
3.97	21.8	1.73	9,900
4.77	30.3	1.66	11,900
5.25	42.2	1.90	13,100
5.44	46.1	1.94	13,600
7.08	84.6	2.01	17,700
7.10	80.2	1.85	17,700
8.67	132.0	2.18	21,700

For convenience, the dimensionless variable $\frac{p}{\rho v^2}$ may be termed the *relative performance*, since it shows the ratio of the differential pressure generated by the nozzle in question, to twice that generated by a Pitot tube. Likewise $\frac{Dv\rho}{\mu}$ may be termed the *generalized speed*, since the speed factor v is commonly the most important of the four, and since any given percentage variation of D, ρ, or μ would have just the same effect on the relative performance as the corresponding variation of speed, which is ordinarily the easiest factor to vary. This variable, $\frac{Dv\rho}{\mu}$, occurs frequently in problems of fluid mechanics where it serves to measure the degree of turbulence in the fluid; hence it has been suggested by Dr. E. Buckingham that the term *turbulence variable* would be of some advantage.

In computing the relative performance and generalized speed the values for speed and for differential pressure given in the first two columns were changed over to c. g. s. units; an arbitrary linear dimension of 1 cm. was taken for D in all cases; the density of the water was taken to be $\rho=1$ gram/cm^3; and its viscosity, since the water was at the temperature of melting ice, was assumed to be $\mu=0.0179$ dyne-sec./cm^2. The values of the relative performance were thus found to range from about one to two (i. e., from twice to four times the differential pressure of a Pitot tube) and those of the generalized speed from about three thousand to twenty thousand. Since these variables are dimensionless, the same numerical values would prevail in any other system of normal units, such as the foot, pound-mass, second system; or the foot, pound-weight, second system.

A generalized speed of 20,000 units (with $D=1$ cm.) corresponds to about 66 miles per hour in air having the standard condition

$$\rho=1.221\times10^{-3}\ \text{g/cm.}^3$$
$$\mu=1.81\times10^{-4}\ \text{g/cm. sec.}$$

this last being the viscosity of air at the standard temperature, 16° C. It corresponds to about 130 miles per hour at 20,000 feet altitude, where the density is half as great if the temperature is unchanged.

The results on the double Venturi (B, fig. 1) were worked up in the same manner and are also plotted in figure 2. Solid black circles represent water channel observations; open circles are for the air observations, which will be described directly. In towing the double Venturi a depth of immersion of 60 cm. was maintained.

It is seen that the viscosity effect (slope of the curve) is more pronounced for the double tube than for the single one; a significant point when taken in conjunction with the fact that it has been the French practice to use the double tube on low speed and the single tube on high speed craft.

The foregoing water channel experiments were made in the winter of 1917–18 with the help of Mr. Bailey Townshend.

The same nozzles were given a wind tunnel calibration at the authors' request under the direction of Dr. A. F. Zahm at the Washington Navy Yard, leading to the results which are plotted for air in figure 2. To illustrate the method of reduction, the following table is given containing the data and results for the French single Venturi:

French single Venturi, in air.

Differential pressure (inches water).	Speed (miles per hour).	$\frac{p}{\rho v^2}$	$\frac{Dv\rho}{\mu}$
1.82	30	1.95	9,230
3.55	40	2.26	12,330
5.50	50	2.24	15,400
8.15	60	2.31	18,500
10.45	70	2.16	21,600

In computing $\frac{p}{\rho v^2}$ and $\frac{Dv\rho}{\mu}$ for air the values taken for density and viscosity were, in c. g. s. units, 1.223×10^{-3} and 1.78×10^{-4}, respectively. The former is the value used for standard density at the Washington Navy Yard tunnel; the latter is the viscosity of air at 10° C., which was assumed to be the actual air temperature. In computing ρv^2 it is not necessary to correct for the departure of actual density in the tunnel from standard density, since the data for speed are based on Pitot tube readings. The values of $\frac{Dv\rho}{\mu}$ on the other hand may be several per cent in error due to this cause, but this correction is not worth going into here, because the empirical values for $\frac{p}{\rho v^2}$ are not appreciably influenced by small changes of $\frac{Dv\rho}{\mu}$; the curves are nearly flat. As before D is arbitrarily taken equal to 1 cm.

The final results are plotted in figure 2, using open circles, and are seen to fall slightly higher up on the diagram than the solid circles for water.

The wind tunnel results on the double Venturi were computed and plotted in the same manner. Here the divergence between air and water results is quite pronounced at the high speed end of the range. This may in part be attributed to an experimental error in the water observations already mentioned; that error is probably negligible, but in the right direction to create the observed difference.

For both the single and double Venturi the relative performance in air is higher than in water; this presumably is due to the great difference in compressibility of the two media; the effect operates qualitatively in the direction suggested by thermodynamic reasoning, but is not so large as might have been expected.

The relative performance of the double Venturi is seen to be from two to three times that of the single tube; the viscosity effect, as judged from the slope of the curve, is also decidedly greater for the double Venturi.

2. EXPERIMENTS AT REDUCED PRESSURE IN A WIND STREAM.

No doubt the most novel feature of the present investigation is the work at reduced pressure in a wind stream, by means of which the conditions at any altitude could be reproduced in the laboratory. This was done by means of a small vacuum wind tunnel, which was placed at the authors' disposal by Dr. H. C. Dickinson. The apparatus consisted of an air-tight iron tank, containing a high speed Sirocco blower and a wooden box with a working space for the wind stream 8 inches square and 2 feet long. The blower was mounted at the exit end of the channel and driven through a stuffing box by a motor outside. At the entrance end a honeycomb was constructed for the usual purpose, together with a piezometer for determining wind speed. The nozzle under test was mounted in the middle of the working space and connected to a water manometer in the room outside. The static connection of the piezometer unit was an annular series of holes in the walls of the wind channel near the grid of impact openings; a tube led from this point directly through the outer wall of the iron container to one side of a water manometer. The impact pressure grid is similarly connected to the other side of the same manometer. A mercurial barometer for determining the absolute pressure in the wind stream is connected to the static side of that manometer. The entire arrangement is sketched out in figure 3. Temperature was measured roughly by a thermometer located outside of the throat in the returning air stream and viewed through a glass window.

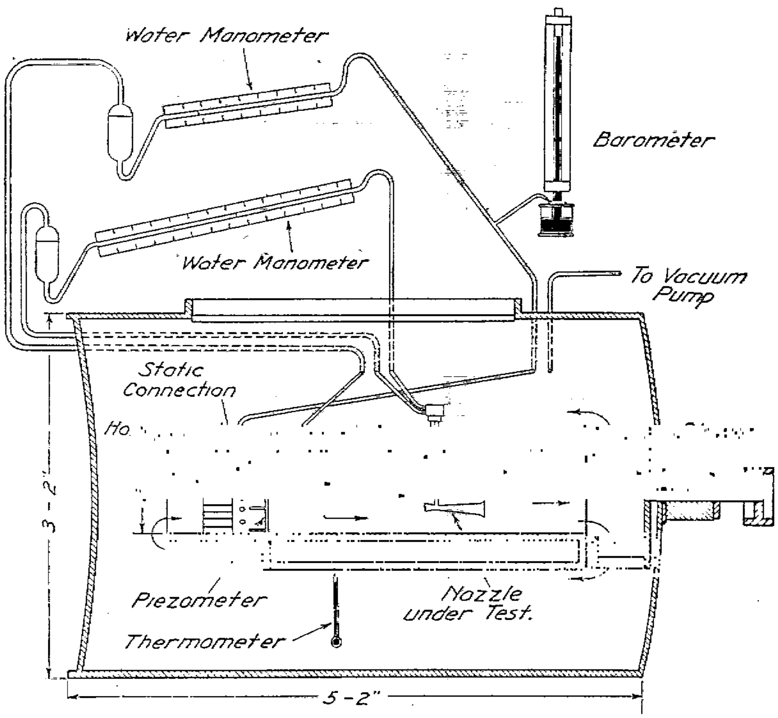


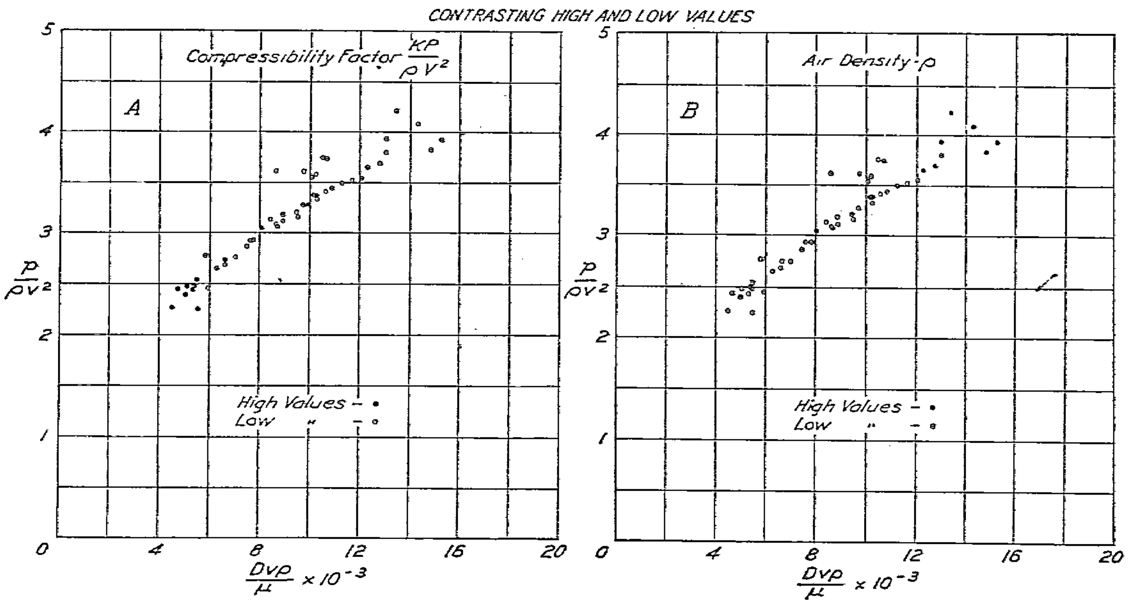
FIG. 3.—Vacuum wind tunnel.

In taking observations the usual procedure was to hold the internal pressure approximately constant by means of the vacuum pump while varying the speed of the blower step by step. This process would then be repeated at a different pressure. Artificial changes of temperature were not undertaken.

The working range of conditions covered was approximately as follows:

Air speed, from 30 to 65 miles per hour;
Pressure, 36 to 76 cms. of mercury;
Temperature, 20° to 28° C.

The final results of the vacuum wind tunnel experiment are plotted with the usual dimensionless variables in figure 4, A nd B. These two diagrams are numerically identical and have been repeated merely to avoid confusion in identifying some of the individual points.



Contrasting high and low values of compressibility factor $\frac{KP}{\rho V^2}$.

High values – • .
Low values – ∘ .

Contrasting high and low values of air density ρ.

High values – • .
Low values – ∘ .

FIG. 4—Vacuum wind tunnel results on United States Army Pitot-Venturi.

Thus in figure 4 A, a contrast has been indicated between points for which the compressibility factor $\frac{\kappa P}{\rho v^2}$ is high, and those for which it is low. Solid black circles are used for high values and open circles for low values. The relative significance of the black and white points is therefore the same in this diagram as it was in the plot for the water channel experiments, figure 2. Evidently there is no correlation between the relative performance and this compressibility variable. The same curve would result from either group of data.

As a matter of interest, although no longer a necessary logical step, figure 4 B has been constructed to show also that there is no correlation between relative performance and density, thus further substantiating the conclusion that the relative performance $\frac{p}{\rho v^2}$ depends solely on the generalized speed $\frac{Dv\rho}{\mu}$. In this diagram the black circles are for high values of the air density ρ, and open circles for low values. Evidently the same curve would be established even if the black points alone, or the white points alone, had been used.

The procedure for putting in the black circles, both in figure 4 A and figure 4 B, was simply to make note of the greatest and least numerical value of the quantity in question—$\frac{\kappa P}{\rho v^2}$, or ρ—and then divide the interval into two equal parts. It was not necessary to specially compute $\frac{\kappa P}{\rho v^2}$ because this is evidently proportional to the ratio of the mercurial barometer reading B to the Pitot pressure h_2 computed from the piezometer reading; actually, this ratio was used instead.

The data from which figure 4 was plotted are given in the following table:

Vacuum wind tunnel data.

1	2	3	4	5	6	7	8	9	10
Run No.	Temp. ° C.	Barom. B mm. merc.	Piezom. h_1 cm. water.	Pit. Vent. h_4 cm. water.	$\frac{Dv\rho}{\mu}\times 10^{-3}$.	Pitot h_2 cm. water.	$\frac{p}{\rho v^2}$	Compress. factor $\frac{B}{h_2}$	Relative density $\frac{\rho}{\rho_0}$
1	20	430	1.30	13.20	8.61	1.83	3.62	235	0.559
	21	430	1.68	16.50	9.76	2.28	3.62	189	.557
	22	428	2.04	20.30	10.70	2.72	3.74	158	.551
2	23	425	0.54	4.32	5.47	0.87	2.48	489	.546
		411	0.57	4.65	5.51	0.92	2.55	449	.527
		407	1.52	13.45	8.93	2.12	3.18	192	.523
		406	1.96	17.73	10.15	2.63	3.38	154	.521
		405	2.10	20.95	10.50	2.80	3.75	145	.519
3	24	393	0.52	4.20	5.13	0.85	2.48	461	.503
		391	0.98	8.13	7.03	1.48	2.77	265	.501
		390	1.49	12.95	8.66	2.09	3.09	186	.499
		390	1.88	16.75	9.75	2.56	3.28	152	.499
		390	2.08	19.95	10.25	2.79	3.58	140	.499
4	24	385	0.52	4.08	5.07	0.85	2.41	452	.492
		385	0.92	7.62	6.63	1.41	2.69	273	.492
		385	1.43	12.70	8.42	2.02	3.14	191	.492
		385	1.81	15.80	9.47	2.47	3.21	157	.492
		385	2.05	19.65	10.10	2.76	3.55	139	.492
5	25	415	0.56	4.45	5.43	0.90	2.50	462	.528
		432	1.06	9.14	7.62	1.56	2.93	277	.551
		489	1.81	16.49	10.62	2.41	3.42	203	.625
		539	1.97	18.29	11.70	2.59	3.53	207	.690
		589	2.01	19.18	12.30	2.62	3.65	225	.752
		643	2.07	21.00	13.00	2.68	3.93	239	.821
		707	2.26	23.90	14.30	2.93	4.09	242	.904
		754	2.28	22.60	14.85	2.95	3.83	255	.962
6	26	620	0.82	7.11	8.03	1.18	3.05	526	.788
		618	1.36	12.32	10.30	1.82	3.38	340	.786
		618	2.08	19.90	12.80	2.69	3.70	229	.786
		618	2.31	25.30	13.45	2.99	4.23	206	.786
		618	2.98	30.50	15.25	3.89	3.93	159	.786
7	28	535	0.66	5.59	6.64	1.02	2.75	528	.676
		535	1.18	10.50	8.90	1.65	3.19	324	.676
		534	1.78	16.25	10.90	2.36	3.46	226	.675
		535	2.17	20.15	12.10	2.84	3.55	189	.676
		531	2.55	25.15	13.05	3.31	3.81	160	.671
8	28	440	0.57	4.07	5.56	0.92	2.25	481	.556
		453	0.99	8.38	7.48	1.46	2.87	310	.570
9	27	436	0.53	4.19	5.38	0.86	2.44	506	.553
		440	1.44	12.58	8.94	2.01	3.13	218	.558
		440	1.91	17.25	10.30	2.57	3.37	172	.558
		440	2.31	21.20	11.30	3.04	3.50	145	.558
10	28	377	0.47	3.81	4.73	0.78	2.45	483	.477
		375	0.85	6.99	6.30	1.31	2.66	286	.475
		373	1.31	11.17	7.80	1.90	2.94	196	.471
		373	1.64	14.10	8.72	2.30	3.07	162	.471
		373	1.96	17.15	9.54	2.66	3.16	140	.471
11	28	369	0.45	3.43	4.54	0.75	2.27	491	.466
		367	0.78	6.10	5.98	1.24	2.47	296	.464
		399	1.68	14.72	5.86	2.66	2.78	150	.504
		408	2.08	18.55	10.30	2.79	3.33	149	.515

In this table columns 1, 2, and 3 are self-explanatory. The piezometer reading h_1 of column 4 is given in cms. of water by reducing the observations taken on an inclined manometer. The Pitot-Venturi head h_4 of column 5 was derived in the same manner. The particular tube investigated was United States Army modified Zahm nozzle No. 30. Values of the generalized speed, $\frac{Dv\rho}{\mu}$, with $D=1$ cm., were computed just as in the case of the French Venturi tubes of figure 2. For the viscosity of air at different temperatures, Sutherland's formula

$$\mu=\mu_0\left(\frac{1+\frac{K}{273}}{1+\frac{K}{\theta}}\right)\sqrt{\frac{\theta}{273}} \tag{29}$$

was referred to, with $\mu_0=17.3\times10^{-5}$ for the viscosity at 0° C in c. g. s. units, and with Sutherland's constant $K=119.4$, θ denoting absolute temperature in degrees centigrade.

The air speed v was obtained from the piezometer reading h_1 at any density ρ by the formula

$$v=C\sqrt{\frac{h_1}{\rho}} \tag{30}$$

in which the coefficient C has the approximate value 54.3. This value was found sufficiently close when determining v for the purpose of computing $\frac{Dv\rho}{\mu}$; but a more exact method was followed for determining v when computing $\frac{p}{\rho v^2}$. The coefficient C is not strictly a constant, but in fact a slowly varying function of $\frac{Dv\rho}{\mu}$, dropping off from about 57.6 to 50.5 while $\frac{Dv\rho}{\mu}$ increases from 5,000 to 20,000. It was determined experimentally by calibration against a Pitot tube in the vacuum wind tunnel, this Pitot tube in turn having been calibrated against a standard Pitot tube in the Bureau of Standards wind tunnel. The results of this experiment were plotted in the form of a curve with $\frac{h_2}{h_1}$ as ordinate against $\frac{Dv\rho}{\mu}$ ($D=1$ cm.) as abscissa where h_2 denotes the reading of a standard Pitot tube. This curve was well determined with a large number of points and gives the ratio needed for converting piezometer readings into the equivalent standard Pitot tube readings which are tabulated in column 7. Since by the standard Pitot tube formula the head in cms. of water is

$$h_2=\tfrac{1}{2}\,\rho\,v^2\times\frac{1}{980}$$

and since from (30) in the same units

$$h_1=\frac{\rho v^2}{C^2},$$

it follows that

$$\frac{h_2}{h_1}=\frac{C^2}{1960}$$

or

$$C=44.3\sqrt{\frac{h_2}{h_1}}$$

Thus the average value 1.5 observed experimentally for the ratio $\frac{h_2}{h_1}$ gives rise to the approximate value 54.3, mentioned above, for the coefficient C. Incidentally it is interesting to note the great difference between the performance constant of the piezometer and that of a Pitot tube, the Pitot head reading for a given air speed in the vacuum wind tunnel being 50 per cent greater than that of the piezometer. This is probably accounted for by the fact that the piezometer integrates the air flow over the cross section, while the Pitot reads the maximum velocity. The further fact that this ratio is not constant but decreases slightly for increasing values of $\frac{Dv\rho}{\mu}$ suggests that the velocity distribution and state of turbulence in the vacuum wind tunnel will appreciably differ for different conditions.

Column 8 shows the relative performance $\frac{p}{\rho v^2}$ for the Pitot-Venturi tube. The pressure p is determined by converting the head h_4 given in column 5 into dynes per square cm. to correspond with the c. g. s. units employed throughout for the density and air speed. The air speed for this column is determined from Formula 30 by using the appropriate value of the coefficient C from the empirical curve. The compressibility factor $\frac{B}{h_2}$ in column 9 is, actually, in millimeters of mercury per centimeter of water. This factor is taken as a substitute for the quantity $\frac{\kappa P}{\rho v_2}$ to which it is proportional. If the ratio $\frac{B}{h_2}$ were given as a dimensionless ratio, for example cms. of water per cm. of water, the values in column 9 would be 1.36 times as large, showing that the impact pressure of the moving air stream in these experiments varied from about 1/650 to 1/165 of an atmosphere. The rarefaction in the throat of the Venturi is about five times as much.

Similarly the relative density $\frac{\rho}{\rho_0}$ is shown in column 10. This varied from about 0.44 up to about 0.91 taking for the standard density, as before, the value 1.221×10^{-3} gms./cm.^{3}, which corresponds to a barometer reading $B_0=760$ mm., and temperature 16° C. These values are computed by dividing the relative pressure $\frac{B}{B_0}$ by the relative absolute temperature $\frac{\theta}{\theta_0}$, in which $\theta_0=289°$ C. absolute.

Inspection of the final plot A or B, figure 4, shows a very pronounced slope, the observed data for relative performance starting far below the normal value 3.2 assumed in the specifications for the instrument, and rising to a value somewhat higher.

Realizing the difficulty of discovering what might happen on an airplane in free flight 10,000 feet above the earth, from observations conducted in a space 8 inches square by 2 feet long, it was originally expected to attach only qualitative significance to the results of the vacuum wind tunnel experiment.

Nevertheless it seemed worth while to determine in what respect the conditions of the experiment differed from the conditions of free flight, so as to judge in which direction, if at all, the observed data would be expected to deviate.

Aside from errors of observation, four fundamental circumstances are worth considering:

(1) The proximity of the walls of the channel to the instrument might disturb the flow; but it is difficult to judge whether this would increase or decrease the performance of the nozzle.

(2) The velocity distribution over the cross section might vary in such a way, when the density and speed of the air are changed, that the actual velocity in the neighborhood of the instrument orifices would fail to bear a constant ratio to the integrated velocity given by the piezometer. In the modified Zahm nozzle the Pitot opening and the upstream Venturi opening are separated by a transverse distance of several inches, or nearly half the diameter of the channel. At low densities or low speeds, where the medium is not so excessively turbulent, the velocity distribution might conceivably be more sharply parabolic than it would at the higher speeds and densities. In this event the relative performance of the nozzle would apparently increase with increasing values of speed and density. However, no direct evidence of an appreciable change in velocity distribution was detected during an extensive series of experiments in which the velocities at different points in the cross section were explored with a Pitot tube. Up to within one inch of either wall, the velocity at any point bore a practically constant ratio of 1.4 to the average integrated velocity. The velocities varied irregularly as much as 6 per cent or 7 per cent above or below this average ratio. This variation, if systematic, should from dimensional considerations be some function of $\frac{Dv\rho}{\mu}$; that is, practically, some function of the product $v\rho$; but the curves plotted in that manner did not show any systematic tendency.

(3) The unsteady state of the flow might give instrument readings perceptably different from those corresponding to a steady state. Some fluctuation of speed was unavoidable. Now it is commonly recognized that the mean reading of a Pitot tube acted on by a rapidly fluctuating current of air is higher than the value which would result from the same speed if actually steady; for the Pitot head is proportional to the square of the speed, but the square root of the mean value of v^2 is greater than the mean value of v itself. If the same reasoning is extended a step further it leads to the conclusion that the mean reading of an instrument actuated by a force proportional to some power of the speed higher than the second will be more greatly augmented by fluctuations than the corresponding reading of an instrument like the Pitot tube acted on by a force actually proportional to the second power of the speed. The plot of the vacuum wind tunnel results does show that the Pitot-Venturi tube generates a differential pressure proportional to a higher power of speed than the second. This explanation would lead us to expect abnormally high readings for the relative performance of the Pitot-Venturi tube as a result of the unsteady state, although it is not apparent just how large the effect would be, and it is doubtless small.

(4) A further source of explanation lies in the excessive degree of turbulence undoubtedly existing in this small apparatus, which was put together from available parts without the usual refinements of a larger wind tunnel. It seems possible that rotating elements of fluid going into the entrance cone of the Venturi might to some extent become straightened out, thus increasing the actual speed of air through the throat beyond the average linear speed of the approaching fluid. If this hypothesis is correct a relatively greater performance should be expected from the Venturi in a turbulent medium than in a uniform medium. Moreover the same condition of turbulence might diminish the piezometer reading due to the impact of eddies against the static openings, thus further accentuating the same effect. The hypothesis regarding turbulence was tested experimentally, to a limited degree, by repeating the observations on a Pitot-Venturi tube in an ordinary wind tunnel with and without a netting across the tunnel in the approaching air stream. A negative result was obtained. The netting appeared to have no effect. This test was not considered conclusive however, because the amount of turbulence created by the netting was probably trifling compared with that really existing in the vacuum wind tunnel.

At all events comparison of the vacuum wind tunnel results with the flight test results and ordinary wind tunnel data given below, indicates that the relative performance shown in figure 4 is numerically greater than would be the case in free flight, although qualitatively correct as regards the effect of variations in speed, density, and viscosity.

The vacuum wind tunnel experiments were carried on with the help of Mr. Howard O. Stearns, while the velocity distribution was observed by Mr. Atherton H. Mears and Mr. W. G. Brombacher.

3. AIRPLANE OBSERVATIONS.

Through courtesy of the Engineering Division of the Air Service at McCook Field, flight tests have been made by one of the authors on the same Pitot-Venturi tube tested in the vacuum wind tunnel. The results are plotted in figure 5, choosing the same variables and the same scale as before. High density points are shown by solid black circles, low density by open circles. The range of conditions experienced was approximately as follows:

Airspeed, from 56 to 126 miles per hour;
Pressure, 43 to 75 cms. of mercury;
Temperature, $-12°$ to $0°$ C.

Thus the densities are somewhat higher and the speeds about twice as high as in the vacuum wind tunnel, so that the range of variables hardly overlaps, although the plane was flown as slow as 56 miles per hour and at altitudes approximating 15,000 feet.

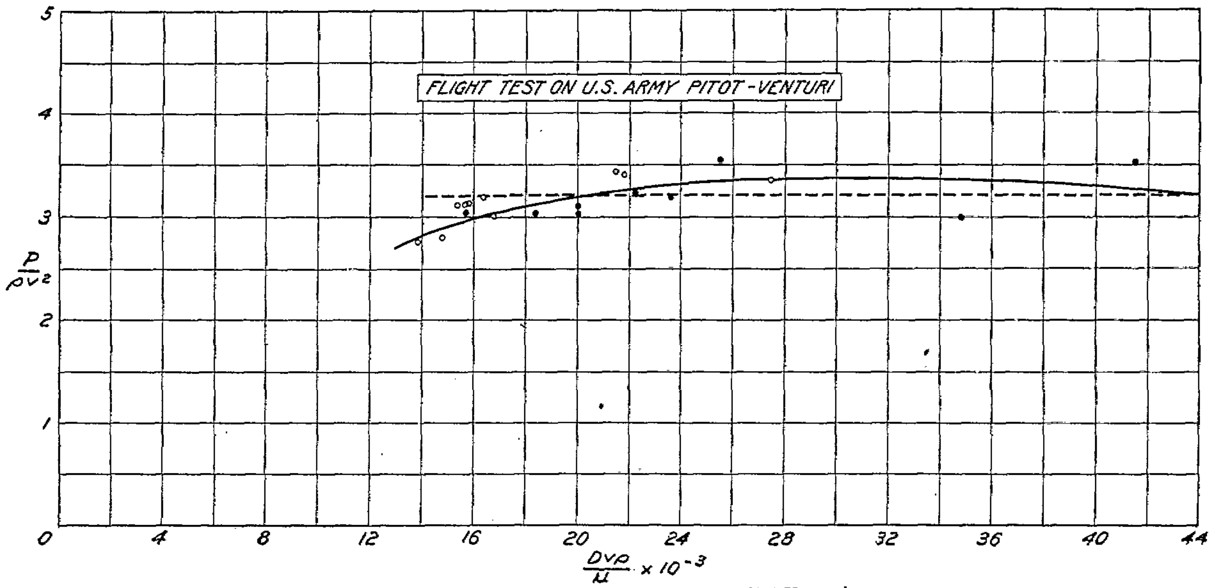


FIG. 5.—Flight test on United States Army Pitot-Venturi.

The data from which figure 5 was plotted are shown, somewhat abridged, in the following table:

Flight test data.

1	2		3	4	5	6	7
Elapsed time (minutes).	Indicated air speed (miles per hour).		$\frac{p}{\rho v^2}$.	Iso-thermal altitude (feet).	Temp. ° C.	Relative density $\frac{\rho}{\rho_0}$.	$\frac{Dv\rho}{\mu} \times 10^{-3}$.
	V_1.	V_2.					
0	0	0	....	240	−8	....	....
2	71.4	71.4	3.18	1,340	−9	1.041	23.6
4	70.5	70.0	3.24	3,100	−6	.966	22.2
6	63.7	64.8	3.10	4,045	−5	.931	20.1
8	65.4	67.4	3.02	5,065	−2	.886	20.1
10	61.3	63.0	3.03	6,175	0	.853	18.4
15	57.8	59.6	3.00	8,545	0	.783	16.8
20	56.9	58.0	3.11	10,710	−4	.726	15.7
24	61.5	61.7	3.18	12,645	−7	.684	16.4
28	60.3	61.0	3.12	14,235	−8	.647	15.8
33	58.8	59.5	3.11	14,690	−11	.642	15.4
38	86.7	83.6	3.44	15,765	−12	.619	21.5
40	49.2	53.0	2.76	15,305	−12	.631	13.9
45	101.8	99.5	3.35	10,030	−5	.746	27.6
47	81.7	78.8	3.41	9,810	−4	.749	21.8
50	50.6	54.0	2.80	10,020	−4	.745	14.9
56	113.5	117.5	2.98	5,075	0	.879	34.8
58	90.4	86.0	3.55	4,850	0	.885	25.5
60	51.5	53.0	3.03	4,955	0	.883	15.7
65	131.2	125.0	3.51	500	−7	1.064	41.5

In this table the indicated air speeds V_1 and V_2 are the readings of a King and Munro air speed indicator, respectively, after purely instrumental corrections have been applied. These corrections were obtained after the flight by direct calibration against standard water columns graduated in miles per hour according to the Zahm and Pitot formulas, respectively. The King air speed indicator was a carefully selected instrument of the standard American Army pattern connected with the modified Zahm nozzle No. 30 under investigation. The Munro indicator was a suitable instrument of British make connected to an R. A. F. Pitot head. During the calibration of the two instruments flight conditions were closely reproduced in the laboratory and corrections determined experimentally for the same readings and instrument temperatures experienced during the flight.

The Pitot-Venturi head was mounted on the left-hand outer strut of the airplane (DH-4), about one-third of the distance down from the upper plane. The Pitot head was mounted in a like position on the right-hand outer strut. The heads were carefully placed so that the distance down from the upper plane was the same in both cases, the object of the flight being to compare the performance of the Pitot-Venturi with a Pitot head rather than to determine absolute air speed. The altimeter and air speed indicators were mounted in a vertical position in the cockpit.

The indicated air speeds V_1 and V_2 are needed for determining the relative performance; the true air speed is computed from the Pitot reading V_2 by making due allowance for the decrease of density at different altitudes; and at the higher altitudes the true air speeds are about 20 per cent greater than the indicated values.

In computing relative performance $\frac{p}{\rho v^2}$, in which p denotes the differential pressure of the Pitot-Venturi nozzle under investigation, ρ the air density and v the true air speed, use is made of the fact that the differential pressure generated by a Pitot tube is one-half ρv^2. The numerical value needed for the relative performance in column 3 is therefore simply one-half the ratio of the head generated by the Pitot-Venturi nozzle to that of a Pitot tube. From the standard formulas

$$h_1=\left(\frac{V_1}{17.88}\right)^2$$

for the Zahm nozzle, and

$$h_2=\left(\frac{V_2}{45.2}\right)^2$$

for the Pitot, it is seen that

$$\frac{p}{\rho v^2}=3.2\left(\frac{V_1}{V_2}\right)^2$$

The isothermal altitude given in column 4 is the ordinary 10° C. altimeter reading corrected for purely instrumental errors by subsequent laboratory comparison under the same pressures and temperatures experienced in flight. The altimeter was set to read 200 feet at the start of the flight, this being approximately the 10° C. altitude corresponding to the actual barometric pressure at the ground.

The air temperatures given in column 5 were observed with a large strut thermometer.

The relative density in column 6 gives as before the ratio of the actual density of the atmosphere to the standard value 1.221×10^{-3} gms./cm.³ The density is figured as before from the barometric pressure and absolute temperature, the pressure in turn being derived from the isothermal altitude of column 4 by reference to the standard 10° C. pressure-altitude table.

Finally, in column 7 values of the generalized speed $\frac{Dv\rho}{\mu}$ are computed (taking the arbitrary linear dimension $D=1$ cm. as before) by reference to the formula

$$v=\frac{44.7\,V_2}{\sqrt{r}}$$

in which r denotes the relative density $\frac{\rho}{\rho_0}$; and by taking the viscosity from Sutherland's formula, as in previous computations. As seen from inspection of the table, the procedure during the flight was to secure a wide variation of speed by diving and straightening out at several different altitudes.

The final plot, as indicated before, shows qualitative agreement with the vacuum wind tunnel observations at the lower speeds and densities while approaching the normal value 3.2 for relative performance (as assumed in the instrument specifications) at the higher speeds and densities; or, strictly speaking, at the higher values of the generalized speed, $\frac{Dv\rho}{\mu}$. Practically the same curve would result had either the high density or low density observations been taken by themselves.

4. ORDINARY WIND TUNNEL DATA.

The same United States Army Pitot-Venturi tube (No. 30) investigated in the vacuum wind tunnel and in free flight had been tested in the 3-foot wind tunnel of the Bureau of Standards. For the purpose of a check on the foregoing experiments the ordinary wind tunnel results, furnished through courtesy of Dr. Lyman J. Briggs, were now recomputed in dimensionless coordinates and have been plotted in figure 6. The results are in close agreement with the flight test and in qualitative agreement with the vacuum wind tunnel results, showing a gradual but pronounced falling off in relative performance toward the lower values of generalized speed.

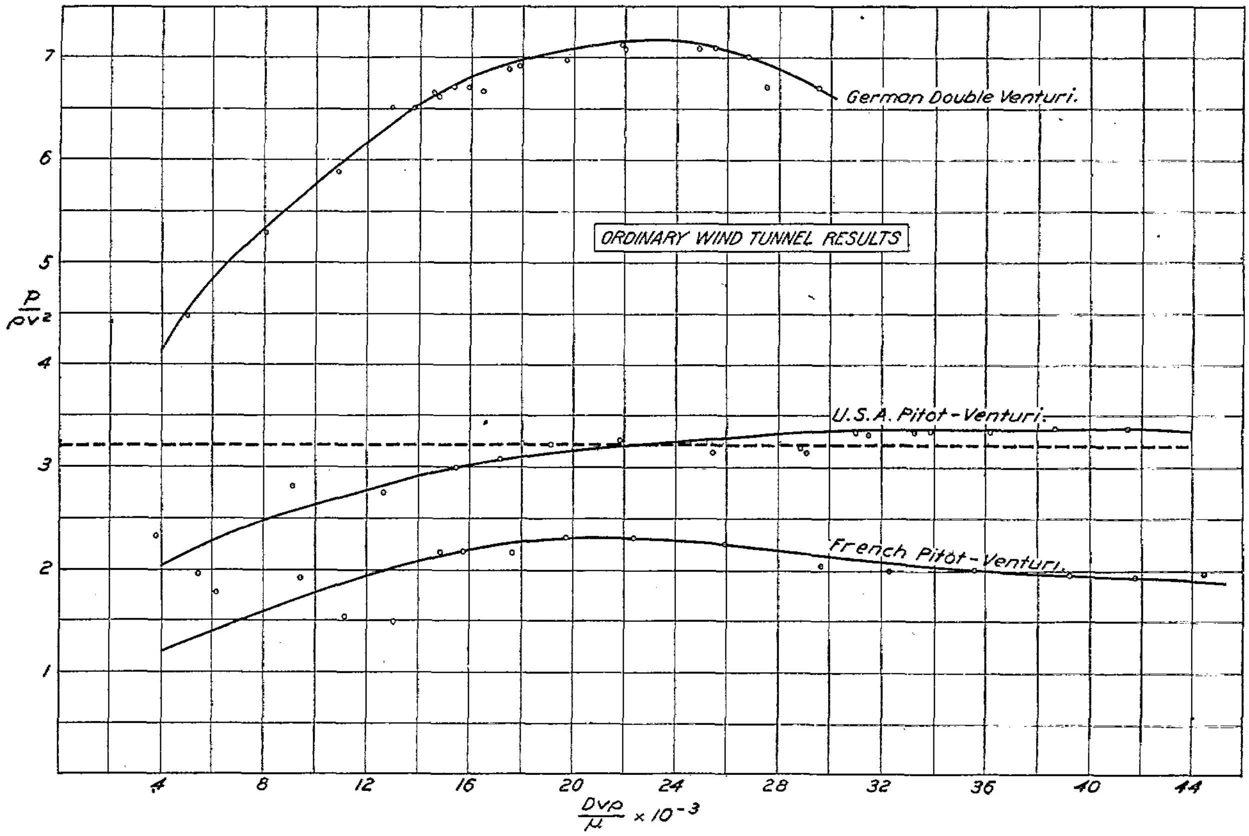


FIG. 6.—Ordinary wind tunnel results.

It is of interest to note the applicability of the dimensionless coordinate diagram to ordinary wind tunnel data with the consequent possibility of inferences regarding the altitude effect, or change in performance at the reduced densities and varying viscosities which may be met at different altitudes. On this account the method has been extended to two other well-known air speed nozzles, the French Pitot-Venturi (Toussaint-Lepère type) and a German double Venturi (Bruhn type), which were tested in the Bureau wind tunnel for the purpose of this investigation. These results also are plotted in figure 6. In all cases the scattering of observations for low values of generalized speed is undoubtedly accidental, due to the very small heads available at the water column under those conditions.

The data for these wind tunnel tests are given in the three accompanying tables. The indicated air speed values in the first column, obtained from the readings of an inclined manometer, afford data for the actual Pitot heads by means of the usual formulas. By comparing this Pitot head with the observed head on the nozzle under test the values of relative performance are derived, and the generalized speed is computed as before.

Ordinary wind tunnel data, United States Army Pitot-Venturi.

Indicated air speed, m. p. h.	Pitot-Venturi head, cm. water.	$\frac{p}{\rho v^2}$	$\frac{Dv\rho}{\mu}\times 10^{-3}$
13.2	1.0	2.32	3.84
19.3	1.8	1.95	5.57
31.4	6.1	2.80	9.15
43.4	12.9	2.74	12.7
53.1	21.1	2.99	15.5
59.0	26.6	3.07	17.2
65.4	34.2	3.21	19.1
75.4	46.1	3.26	21.9
88.1	61.1	3.14	25.7
99.7	77.4	3.14	29.1
108.0	96.0	3.30	31.5
117.6	115.1	3.34	33.9
88.4	60.8	3.14	25.5
100.0	79.4	3.18	28.9
107.2	95.1	3.31	31.0
115.3	110.3	3.32	33.3
125.5	130.9	3.34	36.2
134.0	151.3	3.37	38.7
143.7	173.6	3.38	41.5

Observations.	Approximate temperature, ° C.	Barometer, mm.
First 11	22	747
Last 8	24	747

Ordinary wind tunnel data, French Pitot-Venturi.

Indicated air speed, m. p. h.	Venturi head, cm. water.	$\frac{p}{\rho v^2}$	$\frac{Dv\rho}{\mu}\times 10^{-3}$
21.3	2.0	1.76	6.20
32.7	5.1	1.91	9.53
38.3	5.6	1.53	11.2
45.0	7.5	1.48	13.1
50.8	14.0	2.16	14.9
54.3	16.0	2.16	15.8
60.9	19.9	2.15	17.7
67.9	26.6	2.30	19.8
76.6	33.4	2.28	22.4
89.1	44.6	2.24	26.0
101.9	52.7	2.03	29.7
110.4	60.2	1.98	32.3
121.8	73.6	1.99	35.6
134.9	88.7	1.95	39.3
143.2	99.6	1.94	41.8
152.9	115.2	1.96	44.5

Observations.	Approximate temperature, ° C.	Barometer, mm.
First 8	22	747
Next 5	23	747
Last 3	24	747

Ordinary wind tunnel data, German double venturi.

Indicated air speed, m. p. h.	Venturi head, cm. water.	$\frac{p}{\rho v^2}$	$\frac{Dv\rho}{\mu}\times 10^{-3}$
17.7	3.5	4.47	5.10
28.1	10.5	5.26	8.10
37.6	20.7	5.87	10.9
44.2	31.7	6.5	12.8
47.3	36.2	6.50	13.7
50.1	41.5	6.66	14.5
51.0	43.0	6.60	14.7
53.0	47.1	6.70	15.3
54.9	50.5	6.70	15.8
56.6	53.0	6.66	16.4
60.4	62.6	6.87	17.4
61.3	65.0	6.90	17.8
67.3	87.3	7.70	19.4
67.9	80.3	6.97	19.6
76.0	102.7	7.10	21.9
76.2	102.5	7.07	22.0
86.5	131.8	7.07	24.8
88.2	137.8	7.07	25.5
92.4	150.2	7.00	26.7
101.6	172.5	6.70	29.4

Observations.	Approximate temperature, °C.	Barometer, mm.
1, 2	23	747.3
3, 4, 6, 8, 9, 11, 13, 15	24	747.3
5, 7, 10, 12, 14, 16–20	25	747.3

In this connection the statement made earlier may be recalled with regard to the numerical relation of the generalized speed scale to actual air speed under sea-level conditions; namely, that a generalized speed of 20,000 units, with $D=1$ cm. (approximately the middle of th range)

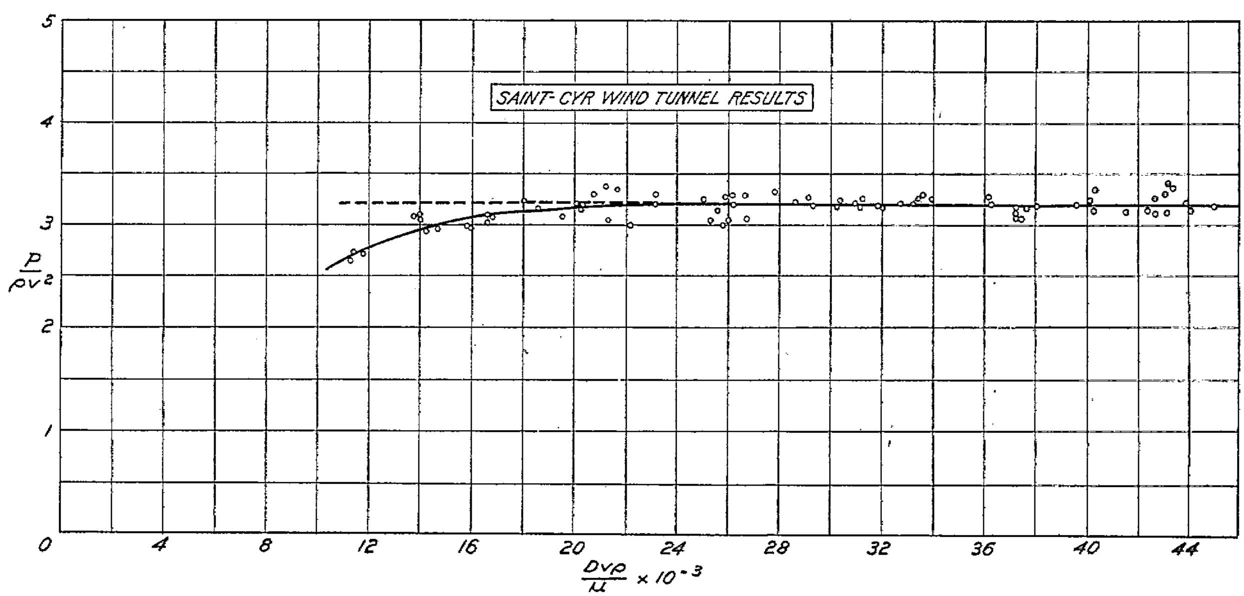


FIG. 7.—Saint-Cyr wind tunnel results.

corresponds to about 66 miles per hour in a sea-level atmosphere. Thus landing speeds, stalling speeds, and speeds of interest in dirigible work lie to the left of the middle, while ordinary airplane speeds are well to the right in figure 6 and similar diagrams.

In connection with the ordinary wind tunnel results, observations which were made in the small instrument wind tunnel at the Aerotechnic Institute at St. Cyr will be of interest, and have been plotted in figure 7. This diagram shows the results of two tests made by Lieut. John A. C. Warner and one of the present authors in November, 1918, on another sample of the small United States Army type Pitot-Venturi tube (No. 23). This investigation was made under the general direction of Maj. George M. Brett, Chief, Airplane Instrument and Testing Division, A. E. F. Air Service, through courtesy of the French Section Technique.

To illustrate the procedure it is sufficient to give the data from one of the two tests, which will be found in the following table:

St. Cyr wind tunnel data, United States Army Pitot-Venturi.

Temperature, 11° C; barometer, 744.5 mm.

Pitot h_1, cm. water.	Pitot-Venturi h_2, cm. water.	$\frac{p}{\rho v^2}$	$\frac{Dv\rho}{\mu}\times 10^{-3}$
2.00	10.7	2.68	11.8
1.85	9.7	2.62	11.3
3.95	23.7	3.00	16.6
4.05	24.8	3.06	16.8
3.70	21.8	2.95	16.0
6.6	40.0	3.03	21.4
6.5	38.9	2.99	22.2
9.6	57.4	2.99	25.8
9.7	58.7	3.02	26.0
10.1	60.7	3.01	26.6
13.2	84.1	3.19	30.3
13.8	88.4	3.20	31.0
13.9	88.1	3.19	31.1
18.8	122.7	3.27	36.2
19.0	121.5	3.20	36.3
20.0	126.7	3.17	37.3
20.4	128.7	3.16	37.7
22.5	144.2	3.21	39.6
23.4	147.0	3.14	40.3
20.8	132.7	3.19	38.1
26.9	168.6	3.14	43.2
26.2	164.5	3.14	42.7
28.0	176.5	3.15	44.1
29.0	184.7	3.18	45.0
25.0	156.7	3.13	41.6
25.8	162.7	3.15	42.4
14.6	93.1	3.19	31.9
14.7	93.1	3.17	32.0
9.2	54.9	3.02	25.3
9.4	56.7	3.12	25.6
5.8	35.7	3.20	20.1
5.5	33.7	3.06	19.6
3.6	21.4	2.97	15.8
3.1	18.3	2.95	14.7

The Pitot head h_1 given by the first column in cm. of water, when compared with the observed head on the Pitot-Venturi in the second column, affords values for the relative performance reported in the third column. In computing the $\frac{Dv\rho}{\mu}$, density and viscosity are derived as usual from the barometer and thermometer readings. The true air speed v was obtained from the Pitot reading by reference to a calibration curve furnished by the Section Technique, showing the result of a comparison between the St. Cyr Pitot and the French standard Pitot at the Eiffel Laboratory.

As in the previous experiment, the relative performance approaches quite closely to the numerical value 3.2 for high values of $\frac{Dv\rho}{\mu}$, but falls off gradually at the lower values.

5. GRAPHICAL COMPARISON OF RESULTS.

The results previously discussed for the performance of five different types of air-speed nozzles in air are brought together for convenient comparison in figure 8. To avoid confusion, the plotted points are left out but all may be seen upon consulting the previous diagrams. The curve shown for the United States Army modified Zahm type of Pitot-Venturi is an average of the flight test and ordinary wind-tunnel results for No. 30. It is seen to agree very closely with the St. Cyr test on the other nozzle, No. 23.

These curves will not be further discussed in the present paper, but evidently merit careful examination by those interested in the details of performance of the various types of air-speed indicator, and provide the necessary experimental basis for inferences regarding the altitude effect.

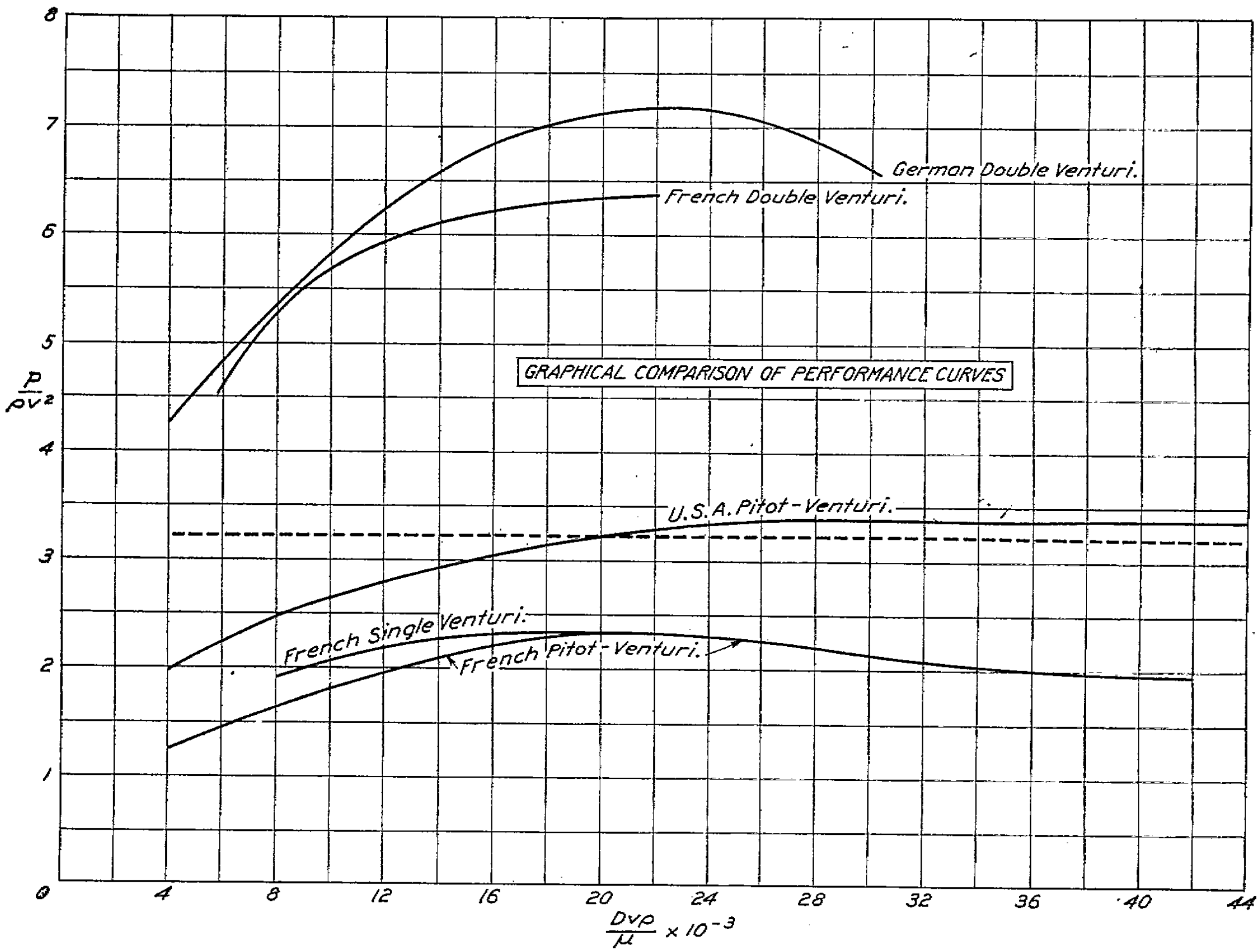


FIG. 8.—Graphical comparison of performance curves.

6. SUGGESTIONS FOR FURTHER INVESTIGATION.

In conclusion, it is suggested that convenient graphical or analytical methods should be developed for computing the altitude correction in practical problems from empirical data such as are afforded by figure 8. Moreover, for the purpose of securing the most exact numerical values, the vacuum wind-tunnel work might well be continued with the use of improved facilities.

Such a program has in fact been begun. A new tunnel with a working space nearly four times the cross section of the previous one is under construction; it is intended to install this tunnel in one of the large altitude chambers of the Bureau of Standards, in which both the temperature and pressure can be controlled over a wider range than before. Finally, additional types of air speed indicator other than Venturi tube should be investigated, verifying the observations with reference to several sample instruments of each type. The laboratory experiments should also be closely paralleled, so far as practicable, by observations taken at low speeds in lighter-than-air craft, and at high altitudes in airplanes. This investigation of the altitude effect is primarily of importance in connection with low-speed or high-altitude flight; for the altitude correction under the conditions of high-speed flight near sea level is sufficiently well given for most instruments by the simple ρv^2 law.

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